Stats 10x Workshop
Basic Maths and Calculator Skills for Statistics
2017

by Leila Boyle

Stats 10x Workshops
The Statistics Department offers workshops and one-to-one/small group assistance for Stats 10x students wanting to improve their statistics skills and understanding of core concepts and topics.

Leila's website for Stats 10x workshop hand-outs and information is here: **www.tinyURL.com/stats-10x**

Resources for this workshop, including pdfs of this hand-out and Leila's scanned slides showing her working for each problem are available here: **www.tinyURL.com/stats-BM**

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Want help with Stats 10x?

STATS 10x appointments
Book your preferred time with Leila here: **www.tinyurl.com/appt-stats**, or contact her directly (see above for her contact details).

STATS 10x Workshops

Workshops are run in a relaxed environment, and allow plenty of time for questions. In fact, this is encouraged ☺

Please make sure you bring your calculator with you to all of these workshops!

- **Preparation at the beginning of the semester:**

Multiple identical sessions of a preparation workshop are run at the beginning of the semester to get students off to a good start – come along to whichever one suits your schedule!

- o Basic Maths and Calculator skills for Statistics **www.tinyURL.com/stats-BM**

- **First half of the semester**

Five theory workshops are held during the first half of the semester:

- o Exploratory Data Analysis **www.tinyurl.com/stats-EDA**
- o Proportions and Proportional Reasoning **www.tinyURL.com/stats-PPR**
- o Observational Studies, Experiments, Polls and Surveys **www.tinyURL.com/stats-OSE**
- o Confidence Intervals: *Means* **www.tinyURL.com/stats-CIM**
- o Confidence Intervals: *Proportions* **www.tinyURL.com/stats-CIP**

Useful Computer Resource:

If you haven't used SPSS before, try working your way through this self-paced tutorial: **www.tinyURL.com/stats-IS**

Second half of the semester

Four theory workshops and one computing workshop are held during the second half of the semester:

- **Statistics Theory Workshops**
  - o Hypothesis Tests: *Proportions* **www.tinyURL.com/stats-HTP**
  - o Hypothesis Tests: *Means* **www.tinyURL.com/stats-HTM**
  - o Chi-Square Tests **www.tinyURL.com/stats-CST**
  - o Regression and Correlation **www.tinyURL.com/stats-RC**
- **Computer Workshop:** Hypothesis Tests in *SPSS* **www.tinyURL.com/stats-HTS**

Why do I need basic maths and calculator skills for Statistics?

Statistics is about summarising, analysing and communicating information. An important part of summarising and analysing information involves having basic numerical literacy skills and doing basic mathematical calculations.

Don't feel you have to do these calculations using mental arithmetic! You can use a calculator to do most of the work for you, but you will need to understand the order of operations for mathematical calculations and be able to apply these rules on your calculator.

Why use a calculator and not a computer?

You won't get access to a computer in your test and exam! However, we do need to check you understand how various formulae should be applied to problems in the test and exam, so you will need to be able to use your calculator to do these.

This stuff is not just useful for Statistics!

You should find these basic maths and calculator skills helpful in your everyday life when you consume numerical information via media reports or the internet. You need to be numerically literate to make informed decisions day-to-day, for example about your personal health or finances, or making judgments about current political issues.

A note about rounding numbers

Often students ask me about rounding numbers – how to do it and how much by.

When you do a calculation, you may end up with an answer that has many (5 or more) decimal places associated with it. People don't deal too well with numbers to this level of accuracy; rounding helps us make a number a little simpler while still keeping its value relatively close to what it was. The result is less accurate, but easier to use, interpret and understand.

How to round numbers

- Decide which is the last digit to keep
- Leave it the same if the next digit is less than 5 (this is called **rounding down**)
- Increase it by 1 if the next digit is 5 or more (this is called **rounding up**)

How many decimal places should I use?

When you are doing calculations in an assignment or test/exam context, you should get some guidance from the question itself. For example, a multi-choice test question may have five answers that are all rounded to one decimal place (1dp for short) so just round your answer when you do the calculation to 1dp.

If you are going to use the value you come up with in a later calculation, go for more accuracy rather than less, as the more you round a number, the more it will affect the results of later calculations that use that value. My default amount of rounding tends to be 4dp – this is reasonably accurate without going over the top!

Exercise 1 – Rounding

1. Write 34.56789 to: → 34.568(3dp) → 34.6(1dp)

(a) 4dp (b) 3dp (c) 2dp (d) 1dp

→ 34.5679(4dp) → 34.57(2dp)

2 Write 43.999 to:

(a) 2dp (b) 1dp → 44.0(1dp)

→ 44.00(2dp)

3. Write 654.34567 to: → 654.346(3dp) → 654.3(1dp)

(a) 4dp (b) 3dp (c) 2dp (d) 1dp

→ 654.3457(4dp) → 654.35(2dp)

What is statistics?

Statistics is the process of collecting data by taking a **sample** from the **population** of interest to draw conclusions (make **statistical inferences**) about the population.

A **population** is the whole group we are interested in.

Examples: the population of all people working full-time in Malaysia

the population of all students studying at the University of Auckland

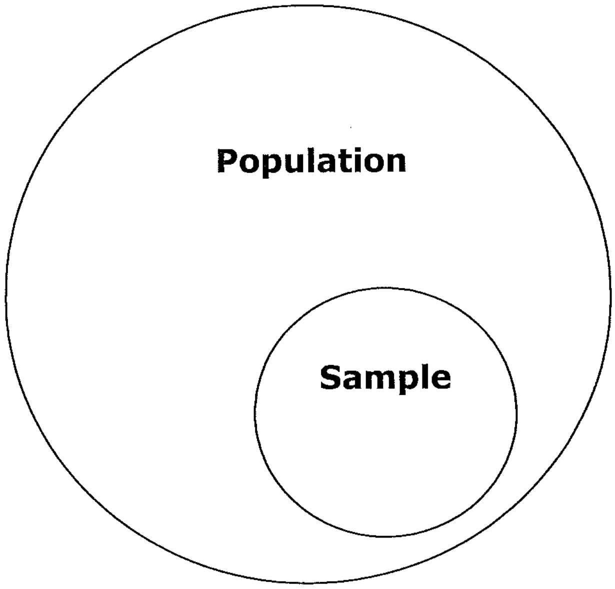
the population of all primary school children in New Zealand.

A **sample** is a portion, or subset, of the population of interest.

Examples: a sample of 350 people working full-time in Malaysia

a sample of 150 students studying at the University of Auckland

a sample of 480 New Zealand primary school children



Why do we sample? It is cheaper, faster and more practical (easier) than taking a census.

A **census** is an attempt to sample the whole population. The next New Zealand-wide census is in March 2018 (the most recent one was in March 2013).

How do we collect our sample data? **Find out more** at the ***Observational Studies, Experiments, Polls and Surveys*** workshop, held in the first half of the semester: **www.tinyURL.com/stats-OSE**

The Language Of Mathematics in Statistics

Mathematical symbols

We can regard a mathematical statement as a sentence in a very simple foreign language.

We use: **numbers** such as 1.2, $\frac{3}{4}$, -7

symbols such as $+, \times, \div, -, ^2$

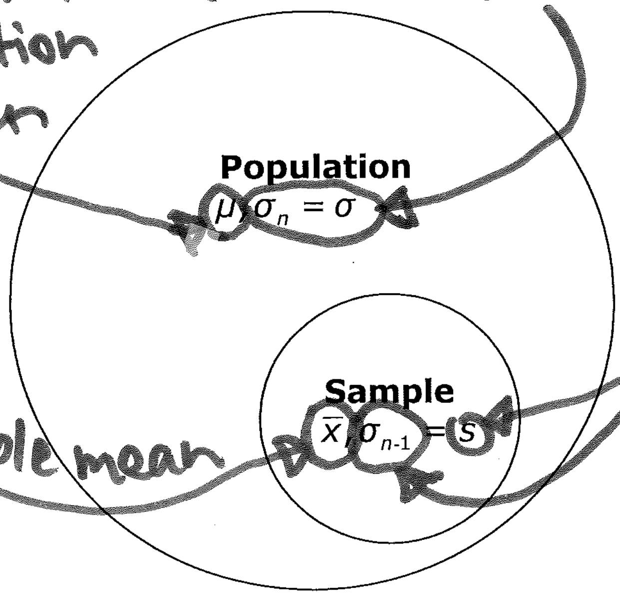
letters for example x, X, y, Y, s

In statistics you may use these special symbols from our Roman alphabet:

- $\bar{x}$ (read as "<u>x bar</u>")
- n

and some letters from the Greek alphabet:

- μ = mu, lower case Greek *m* (pronounced mew)
- σ = sigma, lower case Greek *s*
- χ = chi, lower case Greek *x* (pronounced kie)
- β = beta, lower case Greek *b*
- Σ = Sigma, upper case Greek *S*



Fractions, decimals and percentages

Useful tips and tricks

When dealing with fractions, decimals and percentages, there are a few things you need to keep in mind:

- 'Per cent' means one out of a hundred. We use **percentages** to describe parts of a whole – the whole being made up of a hundred equal parts. Percentages are used frequently in all walks of life.

  For example: *10% off all shoes*

- Decimals are usually numbers that use a decimal point followed by digits that show a value smaller than one. We can also use **proportion** for decimals between 0 and 1.

  For example: *0.1*

- **Decimals** and **percentages** are just different numerical representations of the same value. I find it much easier to understand the value out of 100 compared to the same value out of 1 so I prefer percentages to decimals. However, you will need to be comfortable switching between these two numerical representations!

  - To convert from **decimal** to **percentage**, you **multiply** by 100%.

    For example: *0.1 x 100% = 10%*

  - To convert from **percentage** to **decimal**, you **divide** by 100%.

    For example: *10% ÷ 100% = 0.1*

- Note that 1 = 100% - they are just different representations of the same value!

- When you are asked to find something "of" something else, you are being asked to **multiply** the two numbers together.

  For example: *½ of 4 = ½ x 4 = 2*

- When you are asked to find a percentage of something, you must convert the percentage to a decimal, that is, you need to divide the percentage by 100% before doing the multiplication indicated by the "of".

- When doing fractions on your calculator, you can choose to use the fraction button, or just read the horizontal bar of the fraction as "divided by" (÷).

- When you are dealing with decimals, you can end up with quite small numbers. Your calculator may deal with these by presenting them as "standard form", otherwise known as "scientific notation". This is just your calculator's way of representing a very small number as accurately as it can. You will need to move the decimal point yourself to get it into a decimal number.

For example: *2.459 x 10^-04^ = 0.0002459 [scientific calculator]*
2.459E-04 = 0.0002459 [graphics calculator]
(move the decimal point four places to the left).

.0002.459

Exercise 2 – Fractions, decimals and percentages

Round your answers to 4dp if necessary. "of" means "multiply"!

(a) What is 70% of $7.99?

70% ÷ 100% × $7.99
= $5.59 (2dp)

(b) What is a quarter of a third?

1 ÷ 4 × 1 ÷ 3 (= 1/12)
= .0833 (4dp)

(c) What is $\frac{42}{70}$ as a percentage?

42 ÷ 70 × 100% = 60%

(d) What is $\frac{42}{70}$ as a decimal?

42 ÷ 70 = 0.6

Convert the following numbers to decimals:

(e) 25% = 25% ÷ 100%
= .25

(f) 0.5% = .5% ÷ 100%
= 5×10^-03^
= .005

(g) 5% = 5% ÷ 100%
= .05

(h) 1% = 1% ÷ 100%
= .01

(i) 10% = 10% ÷ 100%
= .1

(j) 2.5% = 2.5% ÷ 100%
= .025

(k) 30% = 30% ÷ 100%
= .3

(l) 100% = 100% ÷ 100%
= 1

(m) What is 0.34 as a percentage?

.34 × 100%
= 34%

(n) What is 0.034 as a percentage?

.034 × 100%
= 3.4%

Getting to know your calculator – useful advice on calculator use

1. I assume you know: [+] [−] [×] [÷]

2. As most scientific and graphics calculators have an in-built order of operations, the equals sign (=) indicates the end of data entry as well as an instruction to calculate up to this point.

2 [+] 3 [=] [×] 4 [=] 20

This key sequence will do $2 + 3 = 5$ then $5 \times 4 = 20$

3. Your aim in using a calculator is to be accurate and to be able to produce the answer **without** having to note figures down on paper part-way through the calculation.

4. Calculators vary. Make sure you understand the **colour** coding on the one you use as each key often has more than one use.

Note: you may have to use one of the following keys:

[2nd Function], [SHIFT] or [INV] key to get the function you want.

5. There are often several ways of doing the same problem. My preferred way of working the problem out is provided in each case.

Pages 10-20 (inclusive) are designed to help you become confident at using your calculator so you can carry out calculations in the following workshops:

First half of the semester

- Confidence Intervals: *Proportions* — **www.tinyURL.com/stats-CIP**
- Confidence Intervals: *Means* — **www.tinyURL.com/stats-CIM**

Second half of the semester

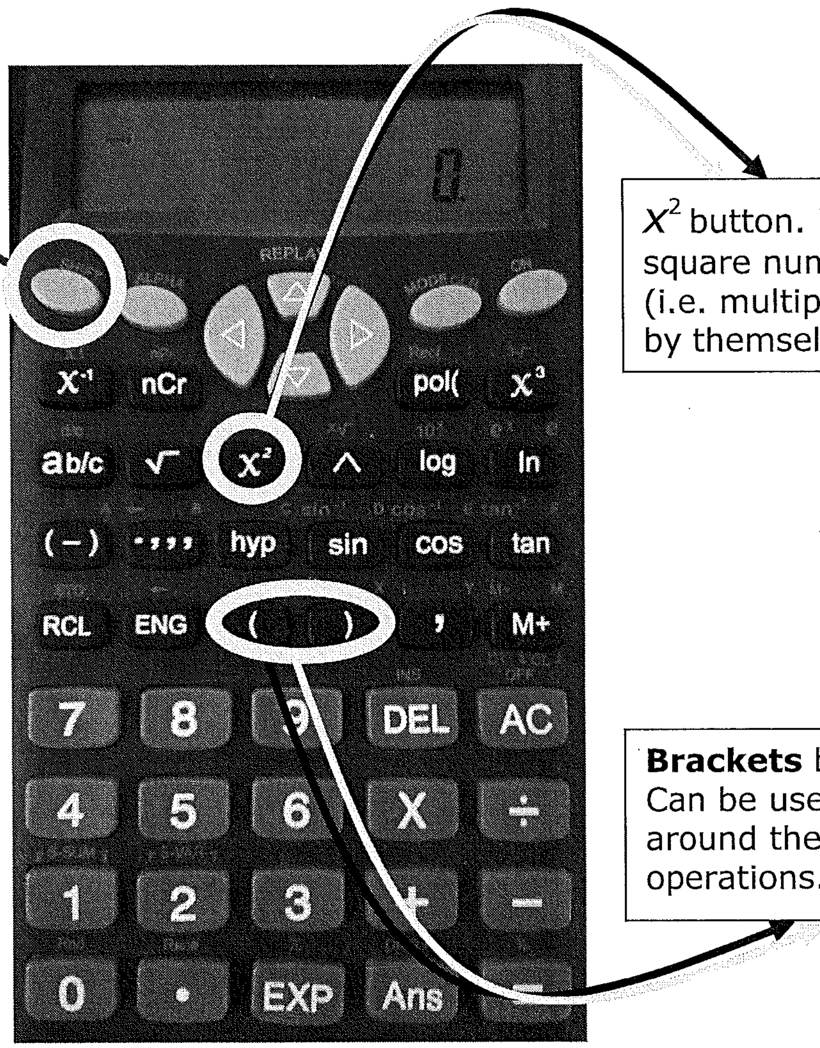
- Hypothesis Tests: *Proportions* — **www.tinyURL.com/stats-HTP**
- Hypothesis Tests: *Means* — **www.tinyURL.com/stats-HTM**
- Chi-Square Tests — **www.tinyURL.com/stats-CST**
- Regression and Correlation — **www.tinyURL.com/stats-RC**

Calculator buttons you will definitely need for Statistics calculations

Here are some useful buttons that you will need to be able to use on your calculator for various statistical calculations:

(and) are keys to open and close brackets respectively. This is very useful where implied (invisible) brackets need to be put in to work around the order of operations.

x^2 this calculates the square of a number (i.e. multiplies it by itself, so $8^2 = 8 \times 8 = 64$).



SHIFT button. Could be called **INV** (inverse) or **2nd Function** on your calculator. Used to access an additional function on any button.

X^2 button. Used to square numbers (i.e. multiply them by themselves).

Brackets buttons. Can be used to work around the order of operations.

Worked examples

Get confident using your calculator by working through the following examples. (Note: you may have to use one of the following keys:

$\boxed{\text{2nd Function}}$, $\boxed{\text{SHIFT}}$ or $\boxed{\text{INV}}$ key to get the function you want).

(a) 6.3^2

6.3 $\boxed{x^2}$ $\boxed{=}$ 39.69

(b) $-8.67 + 5.1$

Either

$\boxed{+/-}$ 8.67 $\boxed{+}$ 5.1 $\boxed{=}$ −3.57

or

$\boxed{(-)}$ 8.67 $\boxed{+}$ 5.1 $\boxed{=}$ −3.57

(c) $\frac{(3.1 + 8.6)}{2.1}$

$\boxed{(}$ 3.1 $\boxed{+}$ 8.6 $\boxed{)}$ $\boxed{\div}$ 2.1 $\boxed{=}$ 5.5714 (4dp)

(d) $6.1 + (3.2 - 2.65)^2$

6.1 $\boxed{+}$ $\boxed{(}$ 3.2 $\boxed{-}$ 2.65 $\boxed{)}$ $\boxed{x^2}$ $\boxed{=}$ 6.4025

(e) $\frac{5.3}{(2.7 + 3.8)}$

5.3 $\boxed{\div}$ $\boxed{(}$ 2.7 $\boxed{+}$ 3.8 $\boxed{)}$ $\boxed{=}$ 0.8154 (4dp)

Do some of these answers seem strange to you? If so, it's because of the **order of operations**, which is built into your calculator.

Order of Operations

In Mathematics we carry out operations such as +, -, ÷, ×, on **numbers** or **expressions**. **Expressions** involve numbers <u>and</u> symbols.

For example, to find:

$7 + 6$	we carry out the operation of addition on the two numbers 7 and 6
8^2	we carry out the operation of finding the square of the single number 8
$(2 - 8)^2$	we carry out the operation of squaring the expression 2 – 8

Calculate $\frac{16}{8}$ means divide the number 16 by the number 8, but $\frac{3+7}{4-1}$ means divide the expression 3 + 7 by the expression 4 - 1.

When each operation is carried out on numbers alone, that is, there are no expressions involved, the order of operations in mathematics is:

When we carry out our calculations, the order in which we do them, that is, the **order of operations** in mathematics is:

Step 1: **Brackets**

If there is more than one set of brackets, work from the innermost one outwards.

Step 2: **Exponents**

This is when a number is to "the power of" another number. This includes squaring, square rooting, etc.

Step 3: **Division or Multiplication**

These operations are tied and are therefore done at the same time, working from left to right.

Step 4: **Addition and Subtraction**

These operations are also tied and done at the same time, working from left to right.

For example to calculate: $(4 + 3^2) - 6 \times 2$

calculate the exponent first	$= (4 + 9) - 6 \times 2$
then simplify the brackets	$= 13 - 6 \times 2$
then carry out the multiplication	$= 13 - 12$
then do the subtraction	$= 1$

use calc!

To describe the four steps, you may have learnt the following the acronym:

Brackets
Exponents
Division
Multiplication
Addition
Subtraction

tied (Division, Multiplication)
tied (Addition, Subtraction)

(O) "of"

B E D M A S

The BEDMAS acronym appears as if division takes precedence over multiplication and addition takes precedence over subtraction. This is not so; remember that division and multiplication are tied so carry out these operations working left to right. Likewise, addition and subtraction are tied, so also carry out these operations working left to right:

To calculate: $8 \times 2 + 16 \div 4$

$8 \times 2 + 16 \div 4$	work from left to right
$= 16 + 16 \div 4$	carry out the multiplication first
$= 16 + 4$	then do the division
$= 20$	finally do the addition

use calc!

To calculate: $7 - 5 + 2 \times 1 - 4$

$7 - 5 + 2 \times 1 - 4$	work from left to right
$= 7 - 5 + 2 - 4$	carry out the multiplication first
$= 7 - 5 + 2 - 4$	then work left to right
$= 0$	then work left to right

use calc!

Doing these calculations by hand is a lot of work but luckily BEDMAS is built into your calculator! All you have to do is type the problem into the calculator exactly as it is written on the page and all the BEDMAS work will be done for you. Try these out on your calculator:

use calc. to do the work for you!

Exercise 3 – Try using your calculator

"of" means "multiply"

(a) $5 + \frac{1}{2}$ of $6 =$ 5 [+] 1 [÷] 2 [×] 6 [=] 8

(b) $7 + 6 \div 2 \times 3 =$ 7 [+] 6 [÷] 2 [×] 3 [=] 16

(c) $-4 \times [(3+7) \div 2] + 30 =$ [(−)] 4 [×] [(] [(] 3 [+] 7 [)] [÷] 2 [)] [+] 30 [=] 10

(d) $\left(\frac{6 + 3 \times (5 - 7)}{10^2}\right)$ [(] 6 [+] 3 [×] [(] 5 [−] 7 [)] [)] [÷] 10 [x^2] [=] 0

(e) $\frac{10}{(10 + 10)}$ 10 [÷] [(] 10 [+] 10 [)] [=] 0.5

Invisible brackets

While your calculator had BEDMAS built into it, you will need to insert some invisible or implicit brackets when calculating expressions. This is because when operations are carried out on expressions, the expression must be simplified first to a single number (i.e. calculated out in full), then the order of operations can be carried out as above.

Consider this problem: $\frac{(8+2)}{(2+3)}$

You probably know that the long line means divide. So this says divide the numerator (the part above the line) by the denominator (the part below the line).

However, we cannot do the division first as we are dividing an expression by an expression.

We can do the addition in the numerator:	8 + 2	= 10
and the addition in the denominator:	2 + 3	= 5
Now we can do the division:	$\frac{10}{5}$	= 2

We could also write the problem like this:

(8 + 2) ÷ (2 + 3) = 2

So, first look for operations on expressions.

Treat any **expressions** as though they are enclosed in **invisible brackets** and simplify them. Carry out operations on **numbers only** using the usual order of operations.

use calc to do this work for you!

Evaluate each of the following using the key indicated.

Note that you may have to insert invisible brackets into some, to all, of the calculations in Exercises 4, 5 and 6.

Exercise 4 | +/– | | (–) | key

(put into your calculator just as they are written!)

(a) −6.4 + 3.8 = −2.6

(b) −7.3 − 5.16 = −12.46

(c) − 9.82 x 6.43 = −63.1426

(d) −71.6 + 8.4 x −2.6 = −93.44

(e) −36.9 + 47.63 = 10.73

Exercise 5 **Bracket keys**

(type into calc as written!)

(a) (4.6 + 5.2) x 3.5 = 34.3

(b) (45.3 − 21.7) ÷ 0.72 = 32.7778 (4dp)

(c) 26.3 x 2.8 x (4.9 − 3.75) = 84.686

(d) $\frac{(61.3 + 17.2)}{86.5}$ = [(] 61.3 [+] 17.2 [)] [÷] 86.5 [=] .9075 (4dp)

(e) $\frac{76.9}{36.2} + \frac{84.2}{19.7}$ = 76.9 [÷] 36.2 [+] 84.2 [÷] 19.7 [=] 6.3984 (4dp)

(f) $\frac{(76.9 - 7.83)}{(96.1 + 17.8)}$ = [(] 76.9 [−] 7.83 [)] [÷] [(] 96.1 [+] 17.8 [)] [=] .6064 (4dp)

(g) $\frac{(8.15 - 5.93)}{(7.2 \times 9.6)}$ = .0321 (4dp)

[(] 8.15 [−] 5.93 [)] [÷] [(] 7.2 [x] 9.6 [)] [=]

Exercise 6 [x^2] key

(a) $12.2^2 - 5.3 =$ 12.2 [x^2] [−] 5.3 [=] 143.54

(b) $4.2 \times 3.8^2 =$ 4.2 [×] 3.8 [x^2] [=] 60.648

(c) $\frac{79.6^2}{(4.3 - 2.15)} =$ 79.6 [x^2] [÷] [(] 4.3 [−] 2.15 [)] [=] 2947.0512 (4dp)

(d) $(1.2 + 5.78)^2 =$ 48.7204

(e) $(4.2 \times 3.8)^2 =$ 254.7216

(f) $1.2 + 6.9^2 =$ 48.81

(g) $\frac{4.1}{7.9^2} + \frac{4.5^2}{2.6} =$ 7.8542 (4dp)

type as written!

Answers

Exercise 1
(a) 34.5679 (4dp)
(b) 34.568 (3dp)
(c) 34.57 (2dp)
(d) 34.6 (1dp)
(e) 44.00 (2dp)
(f) 44.0 (1dp)
(g) 654.3457 (4dp)
(h) 654.346 (3dp)
(i) 654.35 (2dp)
(j) 654.3 (1dp)

Exercise 2
(a) $5.59
(b) 1/12
= 0.0833 (4dp)
(c) 60%
(d) 0.6
(e) 0.25
(f) 0.005
(g) 0.05
(h) 0.01
(i) 0.1
(j) 0.025
(k) 0.3
(l) 1
(m) 34%
(n) 3.4%

Exercise 3
1) 8
2) 16
3) 10
4) 0
5) ½ = 0.5

Exercise 4
(a) −2.6
(b) −12.46
(c) −63.1426
(d) −93.44
(e) 10.73

Exercise 5
(a) 34.3
(b) 32.7778 (4dp)
(c) 84.686
(d) 0.9075 (4dp)
(e) 6.3984 (4dp)
(f) 0.6064 (4dp)
(g) 0.0321 (4dp)

Exercise 6
(a) 143.54
(b) 60.648
(c) 2947.0512 (4dp)
(d) 48.7204
(e) 254.7216
(f) 48.81
(g) 7.8542 (4dp)